Popular Computing

The world's only magazine devoted to the art of computing.

Volume 8
Number 4
April 1980

85

1	1
2	2
3	6
4	2 4
5	1 2 0
6	7 2 0
7	5 0 4 0
8	4 0 3 2 0
9	3 6 2 8 8 0
10	3 6 2 8 8 0 0
11	3 9 9 1 6 8 0 0
12	4 7 9 0 0 1 6 0 0
13	6 2 2 7 0 2 0 8 0 0
14	8 7 1 7 8 2 9 1 2 0 0
15	1 3 0 7 6 7 4 3 6 8 0 0 0

FACTORIALS: Low-order, non-zero digits

Problem 120 (in issue number 36) called attention to the factorial function shown on the cover, and in particular to the circled numbers--the low-order, non-zero digits. The question was this: does the sequence of those digits repeat; that is, cycle?

The sequence for the first thousand factorials is shown, and it appears that the sequence does NOT cycle. However, there is probably insufficient data from which to reach a conclusion.

So more data is needed. The accompanying flowchart suggests a way in which the data can be extended, using a program written in BASIC. Although the problem involves numbers of extremely high precision (factorial 1000 has 2568 digits), most of them are not needed. To secure the results shown, 350 digits were carried at each stage, with one digit in each term of an array, T. The process of multiplying a 350 digit number by another number, N, is given by the following BASIC subroutine:

```
1000  CC = 0
1010  FØR I = 350 TØ 1 STEP -1
1020  T(I) = N*T(I) + CC
1030  CC = INT(T(I)/10)
1040  T(I) = T(I) - 10*CC
1050  NEXT I
1060  RETURN
```

In generating successive factorials, each time the argument N goes through a multiple of 5, the factorial function gains a zero at its low order end. Each multiple of 25 adds two zeros, and each multiple of 125 adds three zeros, and so on. Thus, up to 1000!, there are 249 low-order zeros, and the T array will have been shifted right 249 times. With the array size set at 350 terms, enough digits are retained to insure that the results are correct. If the data is to be extended, then array T should be enlarged.

Publisher: **Audrey Gruenberger**
Editor: **Fred Gruenberger**
Associate Editors: **David Babcock**, **Irwin Greenwald**, **Patrick Hall**
Contributing Editors: **Richard Andree**, **William C. McGee**, **Thomas R. Parkin**, **Edward Ryan**
Art Director: **John G. Scott**
Business Manager: **Ben Moore**

POPULAR COMPUTING is published monthly at Box 272, Calabasas, California 91302. Subscription rate in the United States is $20.50 per year, or $17.50 if remittance accompanies the order. For Canada and Mexico, add $1.50 per year. For all other countries, add $3.50 per year. Back issues $2.50 each.

12642242888682886824484644846886822242822428662642

24284484666264662648868244846886822242822428662644

48468868222428224286626488682662644484644846224282

24284484666264662648868266264224288868288682448462

24284484666264662648868222428448466626466264886828

86826626444846448462242822428448466626466264886826

62642242888682886824484622428448466626466264886822

24284484666264662648868222428448466626466264886828

86826626444846448462242844846886822242822428662648

86826626444846448462242888682662644484644846224284

48468868222428224286626466264224288868288682448468

86826626444846448462242866264224288868288682448466

62642242888682886824484666264224288868288682448464

48468868222428224286626422428448466626466264886824

48468868222428224286626444846886822242822428662648

86826626444846448462242822428448466626466264886826

62642242888682886824484622428448466626466264886822

24284484666264662648868288682662644484644846224282

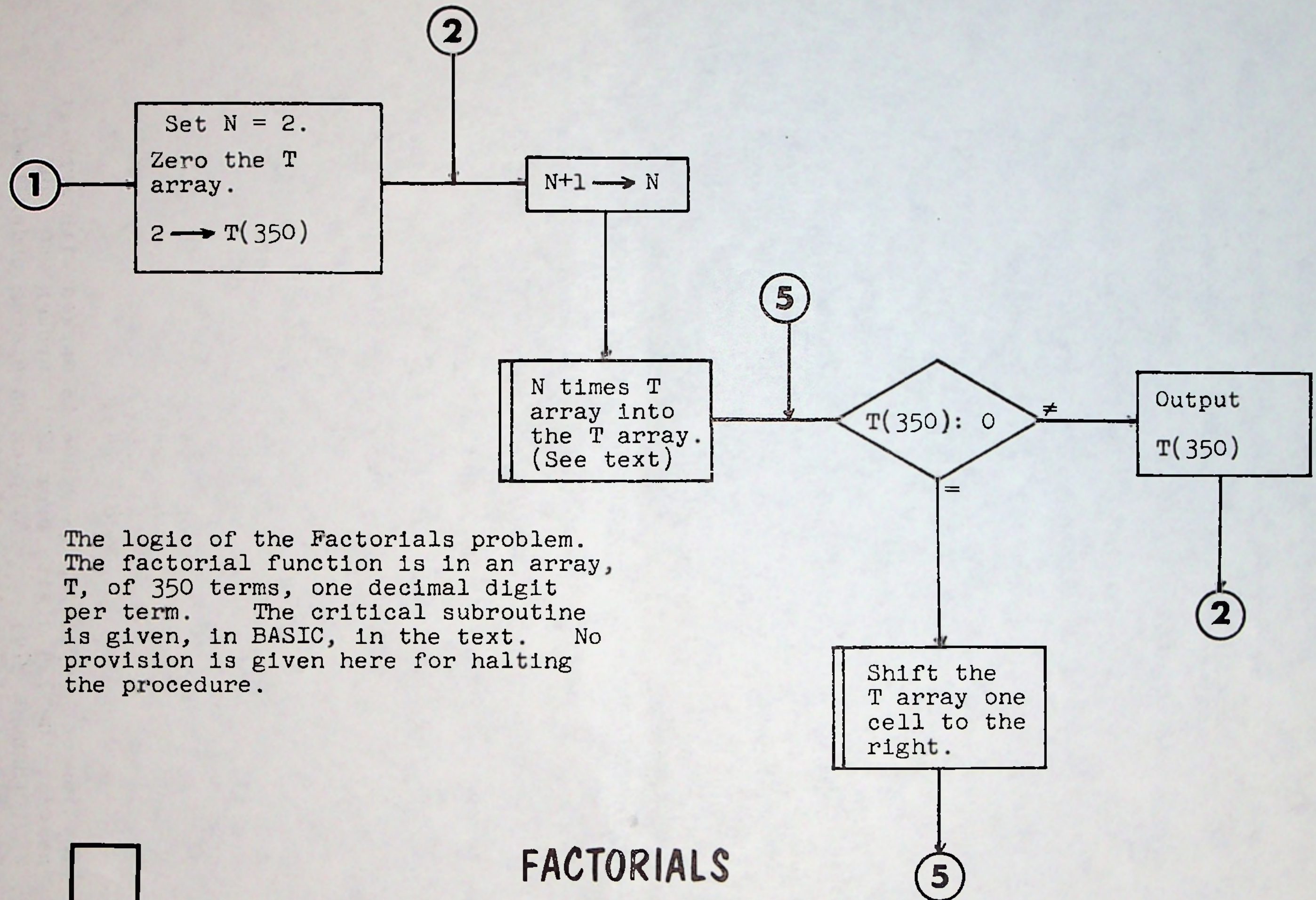
24284484666264662648868266264224288868288682448462

24284484666264662648868222428448466626466264886822

The single, low-order, non-zero digits of the first thousand factorials. Except for 1!, these digits are all 2, 4, 6, or 8, and they distribute themselves as follows:

2:	248
4:	248
6:	260
8:	243.

The longest repeating sequence is marked with arrows; namely, the 125 digits from 125! through 249! and 250! through 374! Triples have been underlined for ease in following patterns by eye.



The logic of the Factorials problem. The factorial function is in an array, T, of 350 terms, one decimal digit per term. The critical subroutine is given, in BASIC, in the text. No provision is given here for halting the procedure.

□

FACTORIALS

Book Review

How To Solve Problems

by Wayne Wickelgren, W. H. Freeman and Company, 1974, 262 pages, soft cover, $6.95.

Aha! Insight

by Martin Gardner, W. H. Freeman and Company, 1978, 179 pages, soft cover, $6.50.

Are Your Lights On?

by Don Gause and Jerry Weinberg, Ethnotech, Inc., 1977, 157 pages, soft cover, no price given.

More than 20 years ago, George Polya wrote How To Solve It, the classical text on problem solving. It is well written, and makes entertaining reading. In fact, one can read it with relish every year. What it boils down to is a clear explanation of how Polya solved some problems, which does little good for you a year later when you can no longer remember just how neatly he solved them.

There isn't much that can be said in order to try to teach someone else how to solve problems. And this fact is germain to computing inasmuch as a great deal of the computing art is problem solving.

There are a few guidelines, to be sure. Some people are naturally or innately better at solving problems than others, just as some people have a sense of music, or painting, or baseball. Most good problem solvers have had a lot of practice; that is, they have already solved a lot of widely disparate problems. They light up when you present them with a fresh new problem--they like solving problems. But even the expert problem solvers can each recall some problem that completely baffled them at one time.

These three books all deal with some aspect of how to solve problems. Wickelgren's book is deadly serious and attempts to reduce problem solving to logical categories. He uses as illustrations most of the celebrated puzzles of the last century or so (the tower of Hanoi; the brakeman, fireman, and engineer problem; Nim; coin-weighing puzzles; the fox, goose, and corn problem; Instant Insanity; etc.) The classification of sub-problems and types certainly cannot hurt; if nothing else, it gives the problem solver something to do while being baffled. The careful step-by-step analysis of classical problems is interesting, and the steps that are described may just apply to your next problem.

Nevertheless, one gets the feeling that the author laid out solutions to old puzzle problems and then found basic principles that that solution illustrated. For the next formidable problem that the reader meets, the solution must proceed in other directions. Let's leave it at this: Wickelgren's book provides interesting (if not especially entertaining) reading.

Martin Gardner's book, on the other hand, is highly entertaining. The subject is still problem solving, but Gardner is concerned with the aha! reaction--the sort of thing that every computer programmer experiences when he finds that last subtle bug in his program--"Oh, there it is! I should have tested out of the loop on greater than instead of greater than or equal to." Gardner is also intrigued with what L. A. Graham called the "surprise attack in mathematical problems"--the small twist that turns an impossible problem into a trivial one. He is also fond of the sneaky problems ("The little indian was the trashman's daughter") which are of no help whatsoever in learning how to solve problems, except that they tend to make you distrustful of the next problem, since it may also be one of the sneaky ones.

In general, for each of the dozens of problems presented in Gardner's book (many of which are not ancient puzzles), the solution depends on some twist that leads to the aha! reaction--and no amount of study of these solutions will be of any help on the next problem, except to give you more practice in solving more problems.

Don Gause and Gerald Weinberg have now extended all this wisdom to include their experience in problem solving in the systems area (although most of what they say applies to more general situations). The book is conversational, and marked by clever aphorisms like:

> Don't mistake a solution method for a problem definition--especially if it's your own solution method.
>
> Don't solve other people's problems when they can solve them perfectly well themselves.
>
> The trickiest part of certain problems is just recognizing their existence.

and my favorite: Don't bother trying to solve problems for people who don't have a sense of humor.

Unfortunately, the book is seriously degraded by poor typography and amateur drawings. Still, it is yet another contribution to that mysterious chemistry called "problem solving." □

BOOK REVIEW

MICROSOFT BASIC

by Ken Knecht

dilithium Press, 1979, 5 1/2 x 8 1/2, paper cover, 158 pages, $8.95

So maybe we can at least understand the BASIC we have at our disposal which, we are told, was written by the Microsoft Company of Bellevue, Washington.

Unfortunately, that firm has written and sold heaven knows how many versions of Microsoft BASIC, each of them incompatible with all the others, and most rather poorly done.

But the situation here is even worse. The author wrote a book about MITS BASIC, which dates back about four years ago. The book has been updated for TRS-80 BASIC, which is also written by Microsoft. But then wouldn't it be proper to name the book TRS-80 BASIC?

With that point understood, the book can then be rated on its merits. It is quite good, and it inserts some computing lore along with the endless rules of BASIC. As anyone who owns a Radio Shack machine knows, there is Level I and there is Level II, and there is Extended BASIC. The book tries to keep the features of each of these different BASICs straight. Where it must be specific, it refers to a version with 6-digit precision arithmetic.

The index is poor; most of its entries are one-word (or less), such as:

STEP, 38
STR, 63
String, 20, 63
Subscript, 77

so that what you might want to look up (e.g., multiple statements per line) is difficult to find.

The book cover mentions the Apple II and the PET, which is again a deliberately misleading ad for the book.

Old Timer's Quiz

Problem 271 (Old Timer's Quiz) in issue 83 was intended simply to arouse some nostalgic feelings among those who came into computing via punched card (EAM) equipment. Incidentally, the quiz didn't ask for what that stood for--it's Electric Accounting Machines, a cutesy euphemism of IBM.

However, it turns out that a lot of people who thought they were old timers had trouble with some of the questions. So here are some answers.

1. The letters KSNJFL were the equivalent of today's ABCDEF in hexadecimal notation. That particular (and weird) combination of letters was used on the ILLIAC-I. The ILLIAC programmers had fun inventing mnemonics to remember the letters; one of them was King Sized Numbers Just For Laughs.

2. Echo check dates back to the earliest line printers specifically designed for computer use, in which the position of a print wheel could be transmitted back to central storage, to be checked against the character that was intended to be printed. This was done by turning the print wheel into an emitter--see question 17.

3. Progressive digiting is a process for obtaining sums of products by simple addition. The process was devised around 1929 by Mendenhall and Warren at Columbia University. It was widely used until around 1948.

4. Block sorting is an ordering of the sorting process on punched card machines. In card sorting, only one column can be sorted in one machine pass. Therefore, to sort on a 4-digit field, the normal procedure is to sort the units' position first, then the tens' position, and so on, with the thousands' position sorted last. In block sorting, the high order position is sorted first, say into 6 groups. Then each of these groups can be sub-sorted independently. Block sorting thus allows for two or more sorters to work on the same deck, and it gets some of the sorting accomplished much sooner.

5. The Princeton-type machines were among the earliest computers to place two instructions in one word. Thus, JOHNNIAC had a 40-bit word, but the instruction length was only 19 bits. The last commercial machine to use this arrangement was the Philco TRANSAC.

6. Maurice Wilkes usually gets the credit for the invention of index registers (called B-boxes in their early days).

7. The team of Wheeler, Wilkes, and Gill was responsible for many early improvements in computers. Credit their team with the invention of the closed (linked) subroutine.

8. The IBM 650 had a 10-digit word size. An instruction word had a 2-digit op-code, a 4-digit operand address, and a 4-digit next instruction address.

The Librascope LGP-30 had a 30-bit word size.

The IBM 1620 had a 12-digit instruction word size, made up of a 2-digit op-code, and two 5-digit operand addresses. Its data word size was variable, ranging from a minimum of 2 digits to a maximum of the storage capacity of the machine. In a 20,000-digit machine, the instruction:

21 19999 19999

could double a data word of 19,600 digits.

The IBM 701 had a 36-bit word size.

9. The General Electric plant at Louisville, Kentucky, was the first installation to use a computer (a Univac-I) for mundane business tasks like payroll and production control. The first attempts they made at programming those tasks were, to put it kindly, disastrous. General Electric, Univac, and the consulting firm of Arthur Anderson shared the responsibility for what was known as the "Louisville Debacle."

10. Hammerlocks are mechanical devices on the type bars of old-style IBM tabulators (e.g., Type 405, 416) that suppress printing. The short hammerlocks prevent printing unconditionally; the long hammerlocks are controlled by plugboard conditions.

11. "Offset Master gang punching" refers to a master/detail deck setup in which information is to be ganged from each master (X-punched) card into all the detail (No-X) cards that follow it, but not on a direct column-for-column basis. Thus, for example, the information in columns 10-19 of the master card is to be ganged into columns 15-24 of the detail cards. The offset requires the use of selectors.

12. "Plug to C" is a plugboard exit that provides a controlled Constant impulse. On a collator, for example, wiring from Plug to C to Primary Feed will cause a card to be read on every cycle.

13. Skip bars were an integral part of pre-026 keypunches, to perform the tabular functions now done by the drum card. The skip bars, of metal or plastic, were notched to indicate card positions to be skipped to.

14. A hopper stop switch allowed for selective disabling of the last-card lever in a feed hopper, so that the last card of the deck could be processed (as opposed to halting the machine in anticipation of adding more cards.)

15. Computers with variable word lengths in some form or other included the IBM 1620, 1401, 702, 705, 7080, and the RCA 301.

16. The Table Lookup op-code on the IBM 650 (operation code 84) allowed for searching a systematic set of arguments until a greater-than-or-equal condition was met, upon which the address of the sought argument was returned. Thus, it was possible to search long tables with one instruction (and no address modification or indexing). This is precisely why the op-code was never used again on any machine; namely, because it provided a sloppy and lazy way of doing something, and people misused it.

17. An emitter is a device (usually a rotating cam-and-circuit-breaker) that produces identical timed impulses to those produced by reading holes in cards. At least, that is how an emitter appears to the user. Actually, all card-position impulses are initiated by a "CB" (circuit breaker).

18. The sorter collating device stacked cards into the stackers in rotation without regard to what was punched on the cards. Thus, if the collating device was set to 7, then the cards being fed would be stacked into 7 stackers in rotation. The general idea was to un-merge sets of merged decks at high speed. The device's principle use was as a scrambler.

19. Selectors are plugboard devices for switching impulses in one of two directions. A pilot selector could be impulsed on one card cycle and would then switch during the entire next card cycle, after which it would automatically return to normal unless it was picked up again. A co-selector acted in a slave relation to a pilot selector as master, performing its switching in cynchronization with its controlling pilot selector; thus, co-selectors were used primarily to expand the number of paths that were made available to be switched. Latch selectors, when in the "transferred" state, would stay that way until they were dropped out in a positive way by a dropout impulse. Digit selectors are a distributing device; that is, an impulse entering a central hub was made available at one of 12 exit hubs. Thus, for example, if a card punched with a 3 in a certain column was to dictate some action, the 3 could be isolated from any other punches in that column by wiring through a digit selector.

20. A back circuit is a condition caused by overlooking the fact that wires carry current in both directions. Complicated plugboard wiring (in which the wirer had it clearly in mind that the impulses went thataway) led to cross connections in which impulses were actually traveling the other way, leading to curious and unwanted machine actions. □

Answer to Last Month's Challenge Problem:

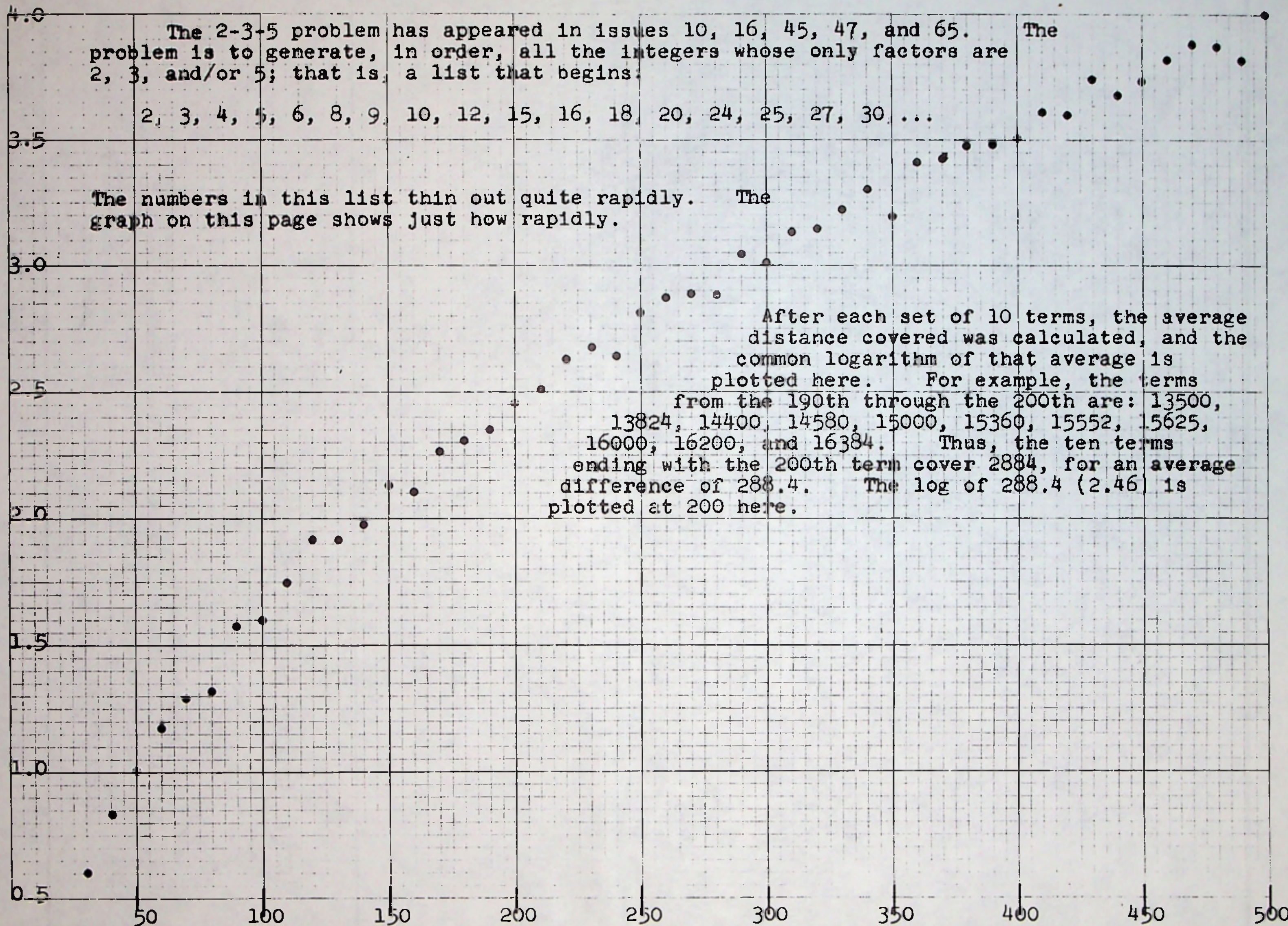
Each player realizes that each of his opponents faces a symmetric situation. Player A, therefore, plays his blue piece <u>left</u>, assuming that Player B will then have to counter with red <u>right</u>, thus blocking Player C. But C has a third choice, and calmly plays both his red pieces, to establish the 3-point position.

The 2-3-5 problem has appeared in issues 10, 16, 45, 47, and 65. The problem is to generate, in order, all the integers whose only factors are 2, 3, and/or 5; that is, a list that begins:

2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30 ...

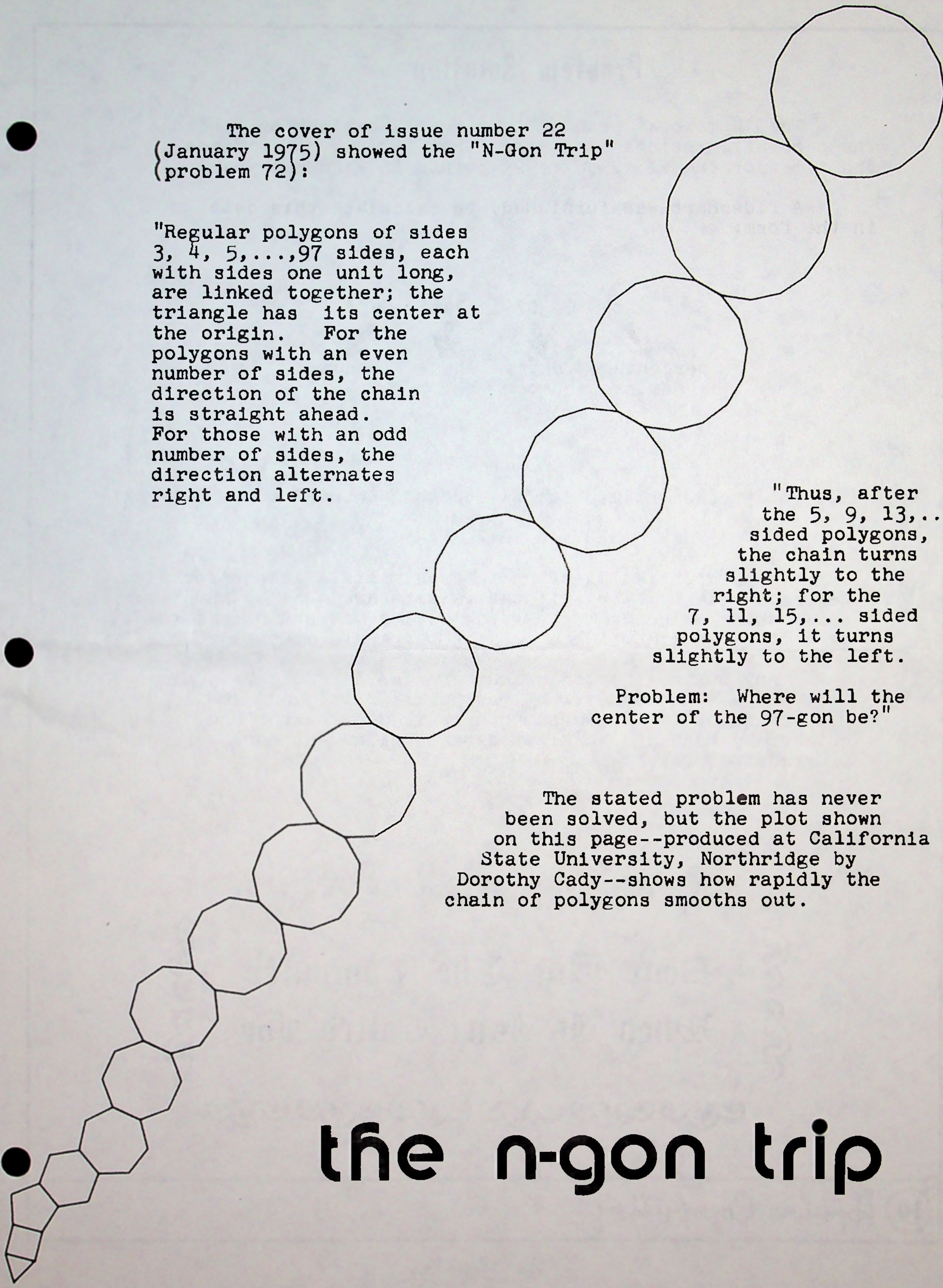
The numbers in this list thin out quite rapidly. The graph on this page shows just how rapidly.

After each set of 10 terms, the average distance covered was calculated, and the common logarithm of that average is plotted here. For example, the terms from the 190th through the 200th are: 13500, 13824, 14400, 14580, 15000, 15360, 15552, 15625, 16000, 16200, and 16384. Thus, the ten terms ending with the 200th term cover 2884, for an average difference of 288.4. The log of 288.4 (2.46) is plotted at 200 here.



The cover of issue number 22 (January 1975) showed the "N-Gon Trip" (problem 72):

"Regular polygons of sides 3, 4, 5,...,97 sides, each with sides one unit long, are linked together; the triangle has its center at the origin. For the polygons with an even number of sides, the direction of the chain is straight ahead. For those with an odd number of sides, the direction alternates right and left.



"Thus, after the 5, 9, 13,.. sided polygons, the chain turns slightly to the right; for the 7, 11, 15,... sided polygons, it turns slightly to the left.

Problem: Where will the center of the 97-gon be?"

The stated problem has never been solved, but the plot shown on this page--produced at California State University, Northridge by Dorothy Cady--shows how rapidly the chain of polygons smooths out.

the n-gon trip

Problem Solution

The "Homework" problem in issue 72 called for finding the percentile points in a year; that is, the exact dates and times of the 1%, 2%,...,99% points in a normal year.

A flowchart was furnished, to calculate this data in the form:

84	307	14	24
percentage number	day number	hour	minute

but the problem called for having this changed to:

NOV 3   2:24 PM.

This might still be an interesting assignment for a beginning class. But if the results are needed, and the whole thing is never to be re-done, then it would seem to be a fine example of when to stop using a computer.

Any business calendar numbers the days of the year (or, a few minutes' work by hand will do it), and the production of the Table shown here is done better and quicker by hand. We might express it as another programming proverb:

Don't Use The Computer
When It Isn't Called For

1	JAN	4	15:36
2	JAN	8	7:12
3	JAN	11	22:48
4	JAN	15	14:24
5	JAN	19	6:00
6	JAN	22	21:36
7	JAN	26	13:12
8	JAN	30	4:48
9	FEB	2	20:24
10	FEB	6	12:00
11	FEB	10	3:36
12	FEB	13	19:12
13	FEB	17	10:48
14	FEB	21	2:24
15	FEB	24	18:00
16	FEB	28	9:36
17	MAR	4	1:12
18	MAR	7	16:48
19	MAR	11	8:24
20	MAR	15	0:00
21	MAR	18	15:36
22	MAR	22	7:12
23	MAR	25	22:48
24	MAR	29	14:24
25	APR	2	6:00
26	APR	5	21:36
27	APR	9	13:12
28	APR	13	4:48
29	APR	16	20:24
30	APR	20	12:00
31	APR	24	3:36
32	APR	27	19:12
33	MAY	1	10:48
34	MAY	5	2:24
35	MAY	8	18:00
36	MAY	12	9:36
37	MAY	16	1:12
38	MAY	19	16:48
39	MAY	23	8:24
40	MAY	27	0:00
41	MAY	30	15:36
42	JUN	3	7:12
43	JUN	6	22:48
44	JUN	10	14:24
45	JUN	14	6:00
46	JUN	17	21:36
47	JUN	21	13:12
48	JUN	25	4:48
49	JUN	28	20:24
50	JUL	2	12:00
51	JUL	6	3:36
52	JUL	9	12:12
53	JUL	13	10:48
54	JUL	17	2:24
55	JUL	20	18:00
56	JUL	24	9:36
57	JUL	28	1:12
58	JUL	31	16:48
59	AUG	4	8:24
60	AUG	8	0:00
61	AUG	11	15:36
62	AUG	15	7:12
63	AUG	18	22:48
64	AUG	22	14:24
65	AUG	26	6:00
66	AUG	29	21:36
67	SEP	2	13:12
68	SEP	6	4:48
69	SEP	9	20:24
70	SEP	13	12:00
71	SEP	17	3:36
72	SEP	20	19:12
73	SEP	24	10:48
74	SEP	28	2:24
75	OCT	1	18:00
76	OCT	5	9:36
77	OCT	9	1:12
78	OCT	12	16:48
79	OCT	16	8:24
80	OCT	20	0:00
81	OCT	23	15:36
82	OCT	27	7:12
83	OCT	30	22:48
84	NOV	3	14:24
85	NOV	7	6:00
86	NOV	10	21:36
87	NOV	14	13:12
88	NOV	18	4:48
89	NOV	21	20:24
90	NOV	25	12:00
91	NOV	29	3:36
92	DEC	2	19:12
93	DEC	6	10:48
94	DEC	10	2:24
95	DEC	13	18:00
96	DEC	17	9:36
97	DEC	21	1:12
98	DEC	24	16:48
99	DEC	28	8:24

The exact times in a normal 365-day year that mark the percentile points. Thus, the middle of the year is noon on July 2.

Prime Checkerboard

The 25 prime numbers that are less than 100 are listed on the facing page.

They would just nicely fill a 5 x 5 array, with the prime 2 at the center because it is so exceptional.

3 is also exceptional. Every other odd prime is either of the form 6K+1 or the form 6K-1. Further, 3 is one of a pair of twin primes, but the other twin, 5, is also twinned to 7.

All this suggests the following problem: arrange the 24 odd primes in one or the other of these patterns:

−	+	−	+	−
+	−	+	−	+
−	+	2	−	+
−	+	−	+	−
+	−	+	−	+

+	−	+	−	+
−	+	−	+	−
+	−	2	+	−
+	−	+	−	+
−	+	−	+	−

where the + and - signs form a modified checkerboard pattern (and 3 is arbitrarily taken as +) according to the 6K±1 rule.

There is one other constraint: the two elements of each pair of twins must lie in cells that are symmetric with respect to the center square. Notice that the ± pattern is such that all pairs of cells symmetric to the center will have opposite signs.

a) Can it be done?

b) Is it a computer problem?

PROBLEM 274

A breakdown of the 25 primes under 100.

2 The only even prime; exceptional here.

3 Not part of either pattern

5	-
7	+
11	-
13	+
17	-
19	+
23	-
29	-
31	+
37	+
41	-
43	+
47	-
53	-
59	-
61	+
67	+
71	-
73	+
79	+
83	-
89	-
97	+

Noting here the nature of each prime, as being either <u>one</u> greater or <u>one</u> less than a multiple of 6.

There are 12 minuses and 11 pluses here.

The 7 sets of twins (excluding 3,5) are marked with squares.

A table of the 1500 consecutive prime numbers after 10,000,000,000. These primes were sifted using the logic shown in issue number 80. Some statistics are given on page 20.

0019	033	061	069	097	103	121	141	147	207	259	277	279	319	343	391	403	469	501	537
0583	589	597	601	631	643	649	667	679	711	723	741	753	793	799	807	877	883	889	949
0963	991	993	999	041	047	051	057	087	101	113	117	153	159	251	371	377	383	411	419
1437	441	467	471	519	533	551	593	623	651	659	677	689	707	723	743	747	797	813	861
1873	899	941	959	969	989	011	037	071	113	143	263	277	323	349	403	407	451	521	527
2553	563	589	641	647	653	671	697	703	707	713	737	743	757	773	781	827	851	907	931
2937	941	959	967	977	983	001	003	019	021	087	099	103	129	141	147	171	187	199	229
3231	267	289	303	337	339	369	441	451	499	523	543	547	577	613	661	667	679	697	711
3723	787	807	859	931	957	033	039	071	089	117	137	153	167	173	227	237	279	297	309
4351	359	407	461	477	527	557	611	633	659	687	701	713	771	797	821	827	851	857	873
4891	897	951	957	139	143	161	163	209	227	283	289	347	361	371	427	467	479	487	551
5553	569	583	611	629	641	661	677	703	773	809	811	839	889	937	973	069	103	117	139
6151	157	187	207	267	279	283	301	307	327	349	351	411	417	433	439	453	469	501	531
6561	591	633	637	727	733	777	787	817	823	829	837	847	889	921	931	949	957	967	979
6997	011	023	027	033	047	077	119	153	197	209	221	243	293	317	327	333	341	371	387
7399	429	441	443	473	491	513	557	569	627	663	669	761	777	837	861	893	921	939	957
7981	043	059	077	089	101	113	121	163	203	217	229	307	341	409	443	467	521	559	581
8593	619	643	661	713	791	803	827	853	857	871	881	911	913	961	983	009	021	031	039
9049	051	081	139	157	169	177	181	189	193	231	247	249	259	267	289	309	333	367	373
9423	427	469	489	519	559	571	589	597	601	639	673	687	703	763	777	783	793	847	931
9937	951	961	973	993	997	071	077	089	117	149	177	201	203	221	227	257	263	291	309
10323	329	333	341	369	381	389	459	483	497	509	551	561	567	591	639	651	681	687	689
10693	747	791	807	819	849	851	857	873	939	947	953	971	017	037	061	079	103	107	121
11137	173	179	187	191	193	211	227	257	293	311	331	377	379	383	407	443	481	487	499
11503	521	529	559	569	577	683	701	731	761	781	803	839	841	883	887	907	929	947	977
11991	997	007	019	027	039	063	073	079	097	157	177	181	223	229	259	301	327	339	409
12411	441	489	507	513	541	543	567	591	621	637	661	699	717	751	769	777	787	807	817
12819	823	831	843	849	859	909	927	961	993	011	029	039	041	059	069	081	123	137	141
13159	167	171	197	213	239	321	323	351	357	369	371	407	423	461	483	497	503	509	539
13561	591	627	677	687	689	707	731	767	783	801	813	851	873	897	911	963	971	981	983
14019	043	061	067	071	097	139	197	223	247	257	301	331	347	389	421	433	463	487	539
14551	569	587	617	623	641	649	653	707	709	749	781	793	797	847	853	863	919	007	031
15033	057	063	139	163	177	183	211	237	261	307	313	321	327	339	357	391	427	429	439
15477	481	499	511	513	517	549	573	577	591	603	639	643	657	679	691	699	733	783	793
15807	811	813	819	847	853	897	901	903	939	957	981	997	003	017	069	083	093	111	113

16129	167	171	183	189	207	219	261	269	281	303	333	339	353	357	413	417	431	443	479
16519	557	569	633	647	671	681	683	687	701	729	737	753	759	773	783	867	927	933	941
16953	963	987	989	031	037	077	089	109	163	173	187	199	227	233	239	241	259	359	361
17367	409	427	499	523	553	569	589	617	637	641	653	683	749	757	773	827	917	919	941
17961	977	997	033	051	081	087	097	151	163	169	207	219	261	271	279	291	319	387	447
18493	513	517	537	559	571	607	621	627	739	771	789	793	813	829	841	879	883	909	933
18939	961	979	981	993	999	011	027	041	053	059	077	093	101	131	147	149	173	177	179
19213	261	297	311	321	329	347	377	417	431	473	483	503	507	509	609	621	629	639	647
19669	693	699	737	747	753	759	767	839	867	891	947	951	957	969	977	987	989	029	049
20097	113	133	137	161	167	203	221	241	263	281	301	307	367	461	467	487	521	529	593
20607	647	649	683	757	769	791	809	817	829	883	911	913	917	007	039	051	063	067	069
21093	103	111	139	177	183	217	249	253	267	279	303	321	327	349	363	393	459	511	517
21523	583	601	643	747	753	769	789	813	847	849	861	901	919	921	973	991	017	029	059
22083	089	093	099	101	141	143	171	173	177	189	209	269	273	287	293	321	369	393	407
22471	479	507	521	531	537	551	573	581	587	647	653	689	707	717	723	807	899	909	917
22941	947	971	017	031	073	091	149	161	167	211	217	239	251	269	277	289	301	353	367
23379	401	403	409	413	419	443	461	491	511	547	563	581	587	599	637	643	653	689	721
23749	751	791	797	823	827	833	853	863	871	877	889	907	911	937	953	967	997	109	127
24129	147	163	193	207	213	261	277	289	301	303	313	327	351	379	399	403	429	439	451
24469	483	493	523	553	583	613	649	651	667	687	693	711	717	751	753	787	799	829	861
24877	907	943	961	973	997	011	027	029	047	053	081	093	183	197	201	237	249	263	327
25333	341	381	389	429	461	471	507	521	561	569	573	579	587	591	597	611	639	647	653
25657	683	687	701	729	771	879	881	897	939	963	977	989	037	079	083	109	137	139	193
26301	307	329	359	373	397	409	451	497	499	527	559	571	607	719	721	727	749	763	791
26847	859	899	919	931	979	013	031	033	039	049	061	139	187	201	241	243	291	307	339
27361	369	381	391	409	459	463	511	519	529	549	559	601	627	637	643	649	679	729	733
27753	819	907	909	987	993	009	023	029	033	039	047	083	119	123	189	191	237	249	261
28317	359	383	399	411	417	431	447	459	477	551	579	593	627	651	683	693	737	777	789
28809	821	893	977	981	031	037	041	059	077	097	101	107	161	163	199	211	223	227	253
29269	287	341	343	421	463	467	469	491	499	509	521	541	607	667	737	773	779	791	811
29857	887	889	899	917	013	043	069	081	097	123	129	133	151	169	213	217	231	237	283
30291	361	373	387	433	471	483	489	511	583	591	639	681	687	697	727	739	741	753	759
30793	813	823	829	837	841	861	867	877	939	967	969	973	979	981	987	003	017	029	051
31081	089	099	123	137	141	161	179	191	201	207	221	231	263	267	291	297	299	347	351
31387	441	473	479	501	521	537	579	603	611	617	627	659	663	687	729	753	777	779	803
31843	879	887	893	903	929	999	013	037	059	083	089	157	187	191	199	223	269	277	283
32323	347	401	433	463	473	521	533	547	569	577	593	599	611	629	703	719	751	773	803
32811	817	827	829	839	923	949	953	003	009	027	037	039	073	079	091	117	139	187	189
33229	297	321	367	387	399	441	453	501	577	579	651	657	669	717	733	741	787	789	793
33873	877	903	921	931	969	021	081	089	123	131	179	201	237	249	251	293	309	333	399

2	14	28	11	54	7	80	0
4	20	30	20	56	3	82	0
6	42	32	6	58	4	84	1
8	18	34	8	60	6	86	0
10	17	36	7	62	1	88	0
12	35	38	3	64	2	90	1
14	20	40	5	66	2	92	2
16	17	42	11	68	1	94	0
18	28	44	4	70	3	96	1
20	10	46	4	72	2	98	0
22	10	48	8	74	0	100	0
24	15	50	5	76	1	102	0
26	15	52	4	78	2	120	2
						182	1

Distribution of the differences between successive primes, for the first 400 primes after 10,000,000,000.

2	10	26	22	50	24	74	14	98	16
4	15	28	16	52	21	76	21	100	16
6	38	30	47	54	29	78	26	102	35
8	12	32	14	56	18	80	24	104	16
10	17	34	17	58	20	82	20	106	15
12	27	36	40	60	46	84	46	108	27
14	19	38	9	62	17	86	12	110	28
16	20	40	17	64	16	88	15	112	20
18	32	42	35	66	42	90	36	114	29
20	18	44	24	68	17	92	15	116	13
22	14	46	21	70	34	94	13	118	14
24	23	48	34	72	30	96	34	120	40

Distribution of all the differences between the 300 primes starting at 10 000 000 019. (Differences greater than 120 were not recorded here.) For example, there are 47 differences of 30 between primes in the range from 10 000 000 019 to 10 000 007 387.